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OPTIMUM DESIGN OF ACTIVELY CONTROLLED STRUCTURES USING COOPERATIVE GAME AND STACKELBERG GAME THEORY

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ABSTRACT

This paper presents an approach for the optimum design of actively controlled structures using a multi-level, multi-objective optimization method to simultaneously address both structural and control design aspects. Structural weight, controlled system energy, controller induced dissipation energy and fast damping time are some of the important objectives to be considered for the integrated design of structural and control systems. A simple approach for dealing with multiple objectives is to convert them to one objective using a weighted combination of objective functions. But in case of a large number of objectives or when the objective functions are not equally important and a hierarchical structure exists, then the problem cannot be solved by the conventional weighted combination approach and therefore multi-level optimization techniques are required. This paper presents a multi-level, multi-objective method for design of actively controlled structures with mixed discrete-continuous variables using a game theory formulation. In particular, the optimization problem is modelled as a cooperative and Stackelberg game. It is assumed that the structural and control objective functions are present at different levels and the exchange of information between the two levels is based on variable updating using response surface methods. It is seen that proposed approach is able to successfully design a controller which brings the structure back to its equilibrium position quickly under the influence of an external disturbance.

KEYWORDS: *Cooperative Game, Multi-level Optimization, Stackelberg Game, Vibration Control*

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INTRODUCTION

Simultaneous optimization of control and structure is the topic of interest to controls and structural engineers. Existing literature contains many different approaches for simultaneous structural and control system optimization but many of the methods that are proposed solve a problem with just one objective function. In cases where a multi-objective optimization is considered, the multi-objective problem is scalarized to a one objective problem using, for example, a weighted combination of objective functions. This scalarization may not work in cases where, for example, the objective functions do not have the same priority and one needs to be solved before other. For such problems where a hierarchical structure exists, multi-level optimization methods are required.

Further, in many of the existing studies, the control system is designed using the linear quadratic regulator (LQR) theory. Since the control cost is directly related with the proper choice of state weighting matrix [Q] and control weighting matrix [R], studies have been done on choosing the optimum values of [Q] and [R] matrices. An LQR design method was presented by Choi and Seo (1999) with stability-robustness properties. A weighted energy method was presented by Ang et al. (2002). AGA based approach for an optimal tracking was proposed by Mansouri and Khaloozadeh (2002). A multi-objective algorithm for optimal selection of weighting matrices for LQR was proposed by Li et al. (2008).

Another design consideration in active vibration control is the determination of number and locations of actuators and sensors on the structure. The optimum placement of actuators has been considered only in the context of control optimization by researchers. The optimum actuator and sensor locations can be found by minimizing a controller performance index as proposed by Demetriou (2000). Anoptimum location of actuators based on the disturbance sensitivity grammian matrix is proposed by Mirza and Van Niekerk (1999).A method based on H_2 norm of the transfer function from disturbance to controlled output to find the optimal locations of sensors and actuators was proposed by Li et al. (2004). An updated approach of maximizing control force and minimizing control energy to ensure good observability and controllability of every mode of structure, has been presented by Braunt and Proslier (2005). An optimum design problem for finding the locations and number of actuator was proposed by Li et al. (2004). It should be noted that all these studies on actuator placement applied to the problem with finding the discrete optimum locations of actuators.

The combined structural/control optimization problem has received little attention as a multi-objective problem with mixed variables (both discrete and continuous). In studies dealing with multiple objectives, a weighted combination of objective functions was optimized. For cases where objective functions have different priorities, multi-level optimization methods are used.

Stackelberg game theory is one of the multi-level methods for dealing with problems with two objectives in a hierarchical structure and is used in this paper. Ghotbi and Dhingra (2013) considered the Nash formulation and cooperative game but the application was limited to very simple objective functions with linear constraints and continuous variables. Two other oligopoly models which are commonly used include the Cournot and Bertrand games. In these oligopoly models, the two players make their moves simultaneously whereas in the Stackelberg approach, the players make their moves sequentially: the leader moves first and based on the leader's choice, the follower selects its strategy.

Ghotbi and Dhingra (2012) proposed a method to calculate the Rational Reaction set (RRS) using sensitivity analysis, Rao et al. (1997) proposed monotonicity analysis to calculate the RRS and Lewis and Mistree (1998) proposed the response surface methods. To construct the RRS, the method used in this work is the response surface method because of the nature of design variables.

Most of the existing methods for simultaneous structural and control optimization work well for structures with few design variables which are largely continuous in nature, and for simple controllers with few gain values. The research done on actuator placement mostly deal with finding the optimum locations of actuators and not finding the optimum number of actuators and the problem is treated only in the context of control optimization. The approach presented in this paper considers a combination of control and structural optimization problem using both control and structure design variables. The optimization problem considered herein is challenging because of the mixed discrete-continuous nature of design variables. Not a lot of literature is available on multi-level and multi-objective optimization problems with continuous and discrete variables. The hierarchical multi-objective optimization with both continuous and discrete variables is solved using Stackelberg as well as Cooperative game theoretic approaches for simultaneously solving both control and structural design problem.

The paper is organized as follows. The design of the control system is discussed in Section 2. Section 3 explains the methods to solve problems with multi-objective functions using game theory methods. The problem formulation is presented in section 4. The method proposed here is applied to a space truss as described in Section 5. Section 6 contains the main conclusions of this paper.

CONTROL SYSTEM DESIGN

The equations of motion are given as:

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = [D]\{f\} \tag{1}$$

Where, $[M]$ $[C]$ and $[K]$ are the mass, damping and stiffness matrices respectively. $[D]$ is the applied force distribution matrix, x is a vector of physical coordinates and f is a control vector

Eq. (1) can be represented in state space form as:

$$\dot{u} = [A]u + [B]f \tag{2}$$

Where $[A]$ is a plant matrix, $[B]$ is an input matrix, {y} is the modal coordinate vector,$u = [\{y\}\{\dot{y}\}]^T$, is a state variable vector and$[\varphi]$ is a modal matrix. The $[A]$ and $[B]$ matrices in Eq. (2) are defined as:

$$[A] = \begin{bmatrix} 0 & I \\ -\omega_i^2 & -2\xi_i\omega_i \end{bmatrix} \tag{3}$$

$$[B] = \begin{bmatrix} [0] \\ \varphi^T D \end{bmatrix} \tag{4}$$

Where ω_i represents natural frequency of the i^{th} mode and ξ_i represents the damping factor of the i^{th} mode. For control design Linear quadratic regulator (LQR) theory is used. The quadratic performance index, J, is minimized to find the optimal control force and is given by:

$$J = \int_0^\infty (f^T[R]f + u^T[Q]u)\,dt \tag{5}$$

Where $[R]$ and $[Q]$ are control and state weighting matrices respectively. The solution to Riccati equation (equation 6 below) is needed to find the minimum value of the performance index which is given in equation 7.

$$[A]^T[P] + [P][A] + [Q] - [P][B][R]^{-1}][B]^T[P] = 0 \tag{6}$$

The minimum value of the quadratic performance index is given as:

$$J^* = u^T(0)[P]u(0) \tag{7}$$

Where $u(0)$ is the initial state vector. As clear from Eq. (7), J^* depends on the initial state which is not fixed and is sometimes unknown. Researchers found that the expected value of J^*, $ev(J^*)$, over a set of possible initial states $u(0)$ is the same as finding the trace of matrix $[P]$.

$$ev(J^*) = trace[P] \tag{8}$$

Eq. (2) can be written in terms of feedback gain matrix as:

$$\dot{u} = ([A] - [B][\kappa])u = [A_{cl}]u \tag{9}$$

Where $[A_{cl}]$ is the closed loop matrix and its Eigen values are defined as:

$$\lambda_i = a_i \pm jb_i \; i = 1......n \tag{10}$$

The closed loop damping ratio ¶$_i$ controls the damping time (settling time) of vibrations and is given as:

$$\xi_i = -\frac{a_i}{\sqrt{a_i^2 + b_i^2}} \; i = 1......n \tag{11}$$

Section 2.1 details the development of a performance index for finding the feedback gains and optimum locations of the actuator.

Actuator Placement

Eq. (2) can be written as:

$$\dot{u} = \begin{bmatrix} [0] & [I] \\ [-\omega_i^2] & [-2\xi_i\omega_i] \end{bmatrix} u - \begin{bmatrix} [0] \\ \varphi^T D \end{bmatrix} \kappa u \tag{12}$$

The energy dissipated by controller is defined as, E_c

$$E_c = \int_0^{\infty} [y \;\; \dot{y}]^T D_c \begin{bmatrix} y \\ \dot{y} \end{bmatrix} dt \tag{13}$$

Where D_c is the damping matrix given as:

$$D_c = \begin{bmatrix} [0] & [0] \\ \varphi^T \, DR^T D^T \, \varphi P_{III} & \varphi^T \, DR^T D^T \, \varphi P_{IV} \end{bmatrix} \tag{14}$$

P_{III} and P_{IV} in the above equation defines the setup of Ricatti matrix as:

$$P = \begin{bmatrix} P_I & P_{II} \\ P_{III} & P_{IV} \end{bmatrix} \tag{15}$$

Eq. (13) can be rewritten as [1]:

$$E_c = [y(0)\dot{y}(0)]^T H \begin{bmatrix} y(0) \\ \dot{y}(0) \end{bmatrix} \tag{16}$$

H in the above equation is the solution to Lyapunov equation which is defined as:

$$[A_{cl}]^T H + H[A_{cl}] = -D_c \tag{17}$$

Eq. (16) is a function of the initial state which again is sometimes unknown. It has been found that maximum of expected value of E_c over a set of possible initial states is the same as maximizing trace of $[H]$ [1]. Therefore,

$$ev[E_c] = trace\,[H] \tag{18}$$

For a combined structural and control design problem, the weight of the structure should be minimized by varying member cross sectional areas with constraints on maximum stresses. Also for control design, trace $[P]$ (Eq. 8) should be minimized and trace $[H]$ (Eq.18) should be maximized using entries of state weighting matrix $[Q]$ and actuator locations as design variables. The procedure outlined next describes how both structural and control objectives can be simultaneously optimized using a game theoretic approach.

GAME THEORETIC METHODS FOR MULTI-OBJECTIVE OPTIMIZATION

In a Multi Objective Optimization (MOO) problem, two or more conflicting objective functions are simultaneously considered. Different methods have been proposed to solve a MOO problem. Game theory is one such approach. There are three different types of game theory methods: (i) Non-cooperative (Nash) game, (ii) cooperative game and (iii) an extensive game as described in sections 3.1-3.3.

Non-Cooperative Game

Each player optimizes his own objective function individually in a non-cooperative game. Other players made some choices and then each player finds its optimum solution. The solutions for each player is called the rational reaction set which is given as:

$$f_1\left(x_1^N, x_2\right) = \min_{x_1 \in X_1} f_1\left(x_1, x_2\right) \rightarrow \quad x_1^N\left(x_2\right) \tag{19}$$

$$f_2\left(x_1, x_2^N\right) = \min_{x_2 \in X_2} f_2\left(x_1, x_2\right) \rightarrow \quad x_2^N\left(x_1\right) \tag{20}$$

Unlike Nash games, where players do not cooperate, there is a method where players cooperate in order to improve the overall solution which is called cooperative game as discussed in the next section.

Cooperative Game Theory

The players cooperate with each other in order to find an optimum solution in a cooperative game. The cooperative model to find a compromise solution using a bargaining function $B(X)$is defined as [2]:

$$B(X) = (u - u^*)(v - v^*)$$
$$= \prod_{i=1}^{2}[U_i(X) - U_i(X_w)] \tag{21}$$

The bargaining function $B(X)$ in case of a multi-objective function problem with p objectives/players is given as:

$$B(X) = \prod_{i=1}^{p}[f_{iw} - f_i(X)] \tag{22}$$

Where f_{iw} is the worst value of the objective function f_i that player i is willing to accept. It is assumed that all objective functions f_i's are equally important. Therefore, the cooperative game theory formulation can be written as:

$$max\ \ B(X) = \prod_{i=1}^{k}[f_{iw} - f_i(X)] \tag{23}$$

In cases, when the objective functions are not equally important, the problem can be divided into different levels. This type of problem is defined as multi-level optimization problem which can be solved using as a Stackelberg game approach which is described in next section.

Extensive Bi-Level Stackelberg Game

The Stackelberg game is used for solving two level problem. Each level has a player (an objective function) and it dominates the player at the second level. The player who dominates is called leader and the other player is called the follower. The leader made the choices and the solution of follower depends on them. The follower optimizes its variables (RRS) based on the values of leader's design variables. Based on the solution of follower, the leader then optimizes its design variables.

The non linear programming (NLP) formulation for a bi-level game is defined as [2]:

Minimize $f_1(l_1, l_2, x)$

by varying l_1

subject to

$(l_2, x) = X_2^R(l_1)$ (24)

Where f_1 is the objective function of the leader and $X_2^R l_1$ is the solution of the following follower problem:

Minimize $f_2(l_1, l_2, x)$

by varying (l_2, x) (25)

Where f_2 is the objective function of the follower. The two level Cooperative and Stackelberg game is shown in Figure 1. One objective function is specified at the leader level and two objective functions are present at the follower

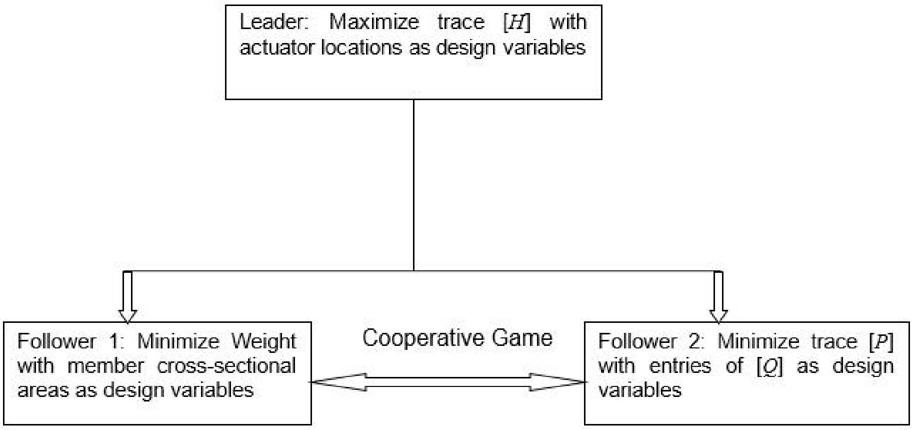

Figure 1: Bi Level Cooperative and Stackelberg Game.

OPTIMIZATION PROBLEM FORMULATION

The multi-objective problem given here is solved as a two-level cooperative and Stackelberg game. The two levels represent the leader and follower. The maximization of energy dissipated by the actuators (trace [H]) is considered as the objective function of the leader. The design variables in this level are the actuator locations. The bargaining function (Eq. 24) is maximized at the follower level which is a compromise between weight minimization and trace P minimization. The design variables for weight minimization problem are the members cross sectional areas whereas the design variables for trace P minimization problem are entries of state weighting matrix.

The information exchange between the two levels is facilitated by the RRS. To find the RRS of the follower problem, optimum solution to follower problem is obtained by using different combinations of leader's design variables. The leader design variables are the actuator locations which are either 1 or 0 (indicating the presence and absence of actuator).The constraints in the problem are handled using a penalty function method which converts a constrained optimization into an unconstrained optimization problem. In particular, exterior penalty function is used which penalize infeasible solutions by reducing the fitness values in proportion to the degrees of constraint violation [16, 20]. This method is used because there is no need to start with a feasible solution with exterior penalty function method and since genetic algorithm (GA) is used as an optimization method in this paper, finding a feasible solution in many GA problems is not always possible. The severeness of the penalty in the penalty function method depends on the value of the penalty factor. A large penalty prevents to search unfeasible region. A very small penalty will tend to spend too much time in searching an unfeasible region [17].A comparison of six penalty function strategies is given in [15]. The solution process is described in Figure. 2.

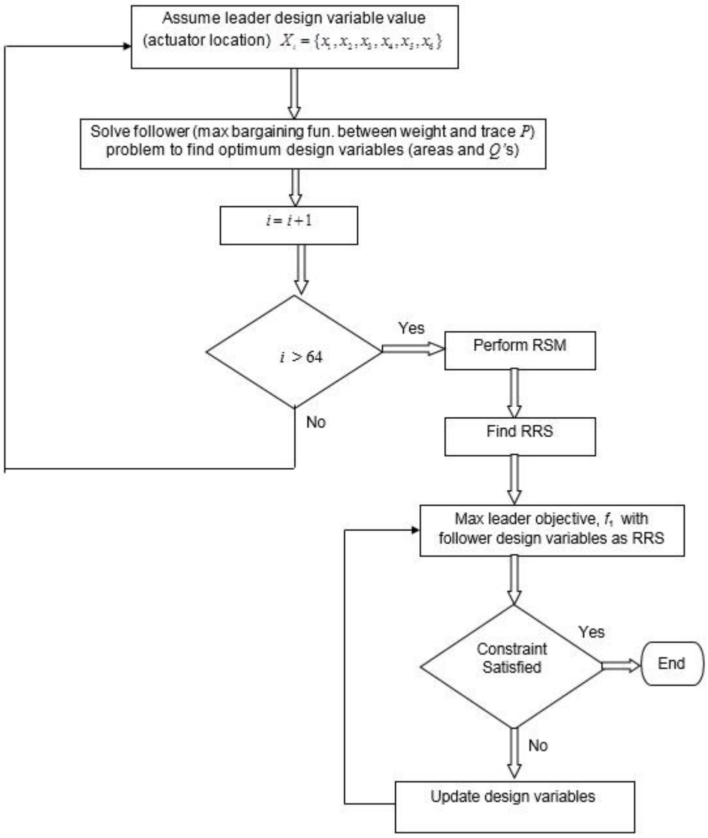

Figure 2: Flow Chart of the Stackelberg Game Solution Process.

DESIGN EXAMPLE

The truss structure shown in Figure. 3 is considered to demonstrate the ideas presented herein for solving multi-level optimization problem. There are ten nodes and twelve elements. The nodal coordinates of the structure are given in Table 1. The edges of the truss have a length of 10 units each and the six legs have a length of $2\sqrt{2}$ units each.

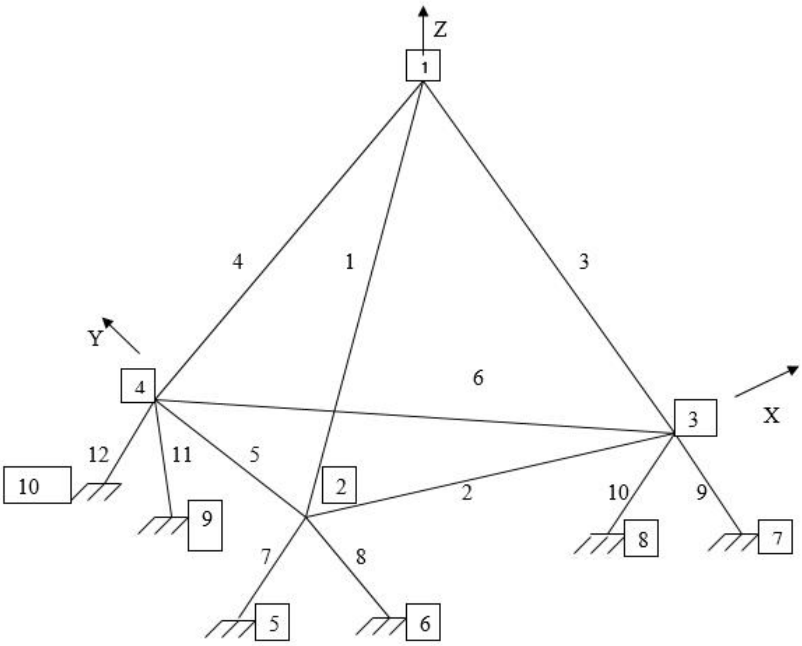

Figure 3: ACOSS Four Structure.

Table 1: Coordinates of the Truss Structure

Node	X	Y	Z
1	0	0	10.165
2	-5	-2.887	2
3	5	-2.887	2
4	0	5.7735	2
5	-6	-1.1547	0
6	-4	-4.6188	0
7	4	-4.6188	0
8	6	-1.1547	0
9	-2	5.7735	0
10	2	5.7735	0

The optimization problem is modelled using Cooperative game and Stackelberg game theory. The three objective functions considered are (i) maximize energy dissipated by the controller, that is, maximize trace [*H*], (ii) minimize weight of the structure, and (iii) minimize the control effort, that is, minimize trace *[P]*. The leader is present at level 1 whose objective is to maximize energy dissipated by the controller by varying the actuator locations in elements 2, 5, 6, 7, 9 and 11. The follower is at level 2 and it contains two objective functions which deal with the weight and the control effort minimization. It is assumed that a cooperative game scenario exists at the follower level. A bargaining function is used to combine these two objective functions at follower level. Constraints are also placed on closed loop Eigen values and damping ratios and are enumerated below.

Leader

Maximize trace [H]

by varying ($x_1 - x_6$)

subject to

$$b_1 > 1.2$$

$$b_2 > 1.5$$

$$\xi_1 > 0.16434 \quad (26)$$

$$Q_{ii} > 1.0$$

$$10 \le A_i \le 2000$$

Follower

Maximize $F_{barg} = \dfrac{(f_{1w} - f_1)(f_{2w} - f_2)}{(f_{1w} - f_{1b})(f_{2w} - f_{2b})}$ (27) by varying ($y_1 - y_{16}$)

Subject to the constraints in Eq. (27). Here f_{1w}, f_{2w}, f_{1b} and f_{2b} denote the worst and best values of weight and trace [*P*].

Stackelberg Solution

An optimum solution for follower variables ($y_1 - y_{16}$) is obtained for each combination of leader design variables $(x_1, x_2, x_3, x_4, x_5, x_6)$. In this case, the response surface regression results in the following RRS for the follower.

$$\begin{aligned}
y_1 &= 173.26 - 14.17x_1 - 22.30x_2 - 21.93x_3 - 28.68x_4 - 30.78x_5 - 40.77x_6 \\
y_2 &= 129.81 + 17.89x_1 - 14.41x_2 - 4.57x_3 - 8.85x_4 - 42.61x_5 - 18.64x_6 \\
y_3 &= 136.17 - 12.81x_1 - 0.15x_2 - 11.15x_3 - 19.05x_4 - 23.44x_5 - 8.88x_6 \\
y_4 &= 132.35 + 6.88x_1 - 12.49x_2 - 3.33x_3 - 6.73x_4 - 20.80x_5 - 11.96x_6 \\
y_5 &= 118.55 + 0.07x_1 - 6.47x_2 - 5.77x_3 - 21.64x_4 - 27.80x_5 - 1.35x_6 \\
y_6 &= 154.07 + 1.79x_1 - 20.94x_2 - 14.54x_3 - 20.70x_4 - 18.87x_5 - 30.68x_6 \\
y_7 &= 98.94 - 44.75x_1 - 3.34x_2 - 12.10x_3 - 24.97x_4 - 11.89x_5 - 8.70x_6 \\
y_8 &= 86.26 - 5.44x_1 - 4.92x_2 - 6.26x_3 - 10.17x_4 - 19.51x_5 - 41.71x_6 \\
y_9 &= 68.00 + 2.07x_1 - 29.91x_2 - 3.41x_3 + 7.41x_4 - 18.94x_5 - 4.87x_6 \\
y_{10} &= 121.66 - 6.07x_1 - 19.89x_2 - 25.67x_3 - 18.61x_4 - 25.11x_5 - 32.36x_6 \\
y_{11} &= 70.51 - 0.61x_1 + 2.00x_2 - 306.61x_3 - 18.06x_4 - 6.04x_5 - 0.60x_6 \\
y_{12} &= 89.33 - 3.51x_1 - 15.27x_2 - 10.40x_3 - 10.80x_4 - 5.83x_5 - 15.55x_6 \\
y_{13} &= 215.66 + 0.54x_1 - 5.98x_2 - 4.91x_3 + 1.18x_4 - 4.43x_5 + 3.36x_6 \\
y_{14} &= 163.58 + 4.32x_1 + 2.20x_2 + 7.74x_3 - 0.84x_4 - 3.51x_5 - 3.58x_6 \\
y_{15} &= 16.45 + 8.54x_1 + 0.43x_2 + 2.11x_3 - 6.29x_4 + 5.12x_5 + 3.98x_6 \\
y_{16} &= 5.69 + 0.77x_1 + 0.41x_2 + 1.83x_3 - 0.03x_4 - 0.38x_5 - 0.61x_6
\end{aligned} \quad (28)$$

Where $y(x)$ approximates the optimum vector which maximizes the bargaining function between weight and trace [*P*] for varying values of $x_1 - x_6$. Note that $y_1 - y_{12}$ are the member cross-sectional areas and $y_{13} - y_{16}$ corresponds to $Q_1 - Q_3$ and Q_{13} By substituting the above RRS into the leader's problem, leader problem is solved to obtain the optimum locations of the actuators. It has been found that only two actuators are needed which should be placed in elements 2 and 6. The maximum value of leader's objective function comes out to be 98.68. The optimum value of follower's objective functions comes out to be 6.98 for the weight of the structure is 6.98 and 1452.4 for trace [P]. The results are listed in Table 2.

The dynamic response of the optimum structure is studied by a unit displacement at node 2 in the *x*-direction at *t*=0. This involves measuring the displacement associated with the line of sight (LOS). The motion of node 1 measures the deviation from the LOS. The LOS error for the optimum design is shown in Figure. 4.

These results can be compared with an earlier work done by Ali et. al (2015) in which the control effort and the effect of changing weighting matrices was not considered. The optimum solution in the earlier work yielded trace [*H*] =151.55 with three actuators located in elements 2, 6 and 11. The weight of the structure was 15.90 and with a LOS error of 2.11. By including the control effort in this work and using optimum values of weighting matrices from controller design, we are able to obtain a 57% reduction in weight with one less actuator and 30 % improvement in LOS error. It may be noted that the earlier design of Ali et al. (2015) gave a better value for trace [*H*], but this improvement was at the expense of an additional actuator, a larger structural weight and a higher LOS error.

Table 2: Design Variables at Optimum Design

Element	Actuator	Areas	*Q*
1		113.79	212.41
2	X	78.34	159.22
3		93.67	15.28
4		104.82	5.28
5		69.11	
6	X	114.49	
7		62.07	
8		56.57	
9		56.47	
10		77.93	
11		46.4	
12		72.69	
Trace H	98.68		
Weight	6.79		
Trace P	1452.4		

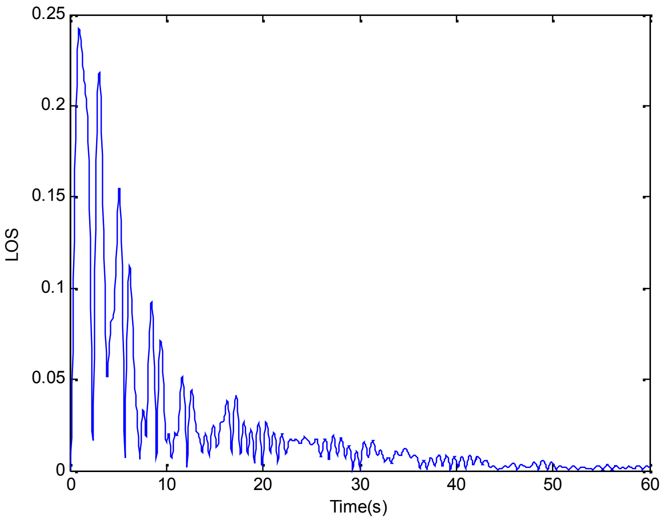

Figure 4: LOS Error for the Optimum Design.

CONCLUSIONS

A multi-level and multi-objective problem for simultaneous structure and control design is solved using a game theory formulation. The optimization problem is modelled as a combination of a cooperative and a Stackelberg game. At the leader level, the energy dissipated by the controller is maximized. At the follower level the structural weight and controller performance index are minimized. The information exchange between the two levels is facilitated by RRS. The problem considered here in consists of a mixture of continuous and discrete design variables and cannot be solved using conventional gradient based optimization techniques. The approach presented in this work is a combination of sequential quadratic programming and genetic algorithms. Therefore, the proposed method can be applied to problems involving mixed discrete and continuous design variables.

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